# MATHEMATICS TEST 

60 Minutes-60 Questions

DIRECTIONS: Solve each problem, choose the correct answer, and then fill in the corresponding oval on your answer document.

Do not linger over problems that take too much time. Solve as many as you can; then return to the others in the time you have left for this test.
You are permitted to use a calculator on this test. You may use your calculator for any problems you choose,
but some of the problems may best be done without using a calculator.
Note: Unless otherwise stated, all of the following should be assumed.

1. Illustrative figures are NOT necessarily drawn to scale.
2. Geometric figures lie in a plane.
3. The word line indicates a straight line.
4. The word average indicates arithmetic mean.
5. Marcus's favorite casserole recipe requires 3 eggs and makes 6 servings. Marcus will modify the recipe by using 5 eggs and increasing all other ingredients in the recipe proportionally. What is the total number of servings the modified recipe will make?
A. 6
B. 8
C. 10
D. 12
E. 15
6. The 35 -member History Club is meeting to choose a student government representative. The members decide that the representative, who will be chosen at random, CANNOT be any of the 3 officers of the club. What is the probability that Hiroko, who is a member of the club but NOT an officer, will be chosen?
F. 0
G. $\frac{4}{35}$
H. $\frac{1}{35}$
J. $\frac{1}{3}$
K. $\frac{1}{32}$
7. For what value of $x$ is the equation $2^{2 x+7}=2^{15}$ true?
A. 2
B. 4
C. 11
D. 16
E. 44
8. Let the function $f$ be defined as $f(x)=5 x^{2}-7(4 x+3)$. What is the value of $f(3)$ ?
F. -18
G. -26
H. -33
J. -60
K. -75
9. A wallet containing 5 five-dollar bills, 7 ten-dollar bills, and 8 twenty-dollar bills is found and returned to its owner. The wallet's owner will reward the finder with 1 bill drawn randomly from the wallet. What is the probability that the bill drawn will be a twentydollar bill?
A. $\frac{1}{20}$
B. $\frac{4}{51}$
C. $\frac{1}{8}$
D. $\frac{2}{5}$
E. $\frac{2}{3}$
10. The ABC Book Club charges a $\$ 40$ monthly fee, plus $\$ 2$ per book read in that month. The Easy Book Club charges a $\$ 35$ monthly fee, plus $\$ 3$ per book read in that month. For each club, how many books must be read in 1 month for the total charges from each club to be equal?
F. 1
G. 4
H. 5
J. 6
K. 75
11. In parallelogram $A B C D$ below, $\overline{A C}$ is a diagonal, the measure of $\angle A B C$ is $40^{\circ}$, and the measure of $\angle A C D$ is $57^{\circ}$. What is the measure of $\angle C A D$ ?
A. $40^{\circ}$
B. $57^{\circ}$
C. $77^{\circ}$
D. $83^{\circ}$
E. $97^{\circ}$

12. When $x=\frac{1}{2}$, what is the value of $\frac{8 x-3}{x}$ ?
F. $\frac{1}{2}$
G. 2
H. $\frac{5}{2}$
J. 5
K. 10
13. In the standard $(x, y)$ coordinate plane, what is the midpoint of the line segment that has endpoints $(3,8)$ and $(1,-4)$ ?
A. $(-2,-12)$
B. $(-1,-6)$
C. $\left(\frac{11}{2},-\frac{3}{2}\right)$
D. $(2,2)$
E. ( 4,-12)
14. The fluctuation of water depth at a pier is shown in the figure below. One of the following values gives the positive difference, in feet, between the greatest water depth and the least water depth shown in this graph. Which value is it?

F. 3
G. 6
H. 9
J. 12
K. 19
15. What is the slope of the line through $(-2,1)$ and $(2,-5)$ in the standard $(x, y)$ coordinate plane?
A. $\frac{3}{2}$
B. 1
C. -1
D. $-\frac{3}{2}$
E. -4
16. In Cherokee County, the fine for speeding is $\$ 17$ for each mile per hour the driver is traveling over the posted speed limit. In Cherokee County, Kirk was fined $\$ 221$ for speeding on a road with a posted speed limit of 30 mph . Kirk was fined for traveling at what speed, in miles per hour?
F. 13
G. 17
H. 43
J. 47
K. 60
17. What is the sum of the solutions of the 2 equations below?

$$
\begin{array}{r}
8 x=12 \\
2 y+10=22
\end{array}
$$

A. $2 \frac{2}{5}$
B. $7 \frac{1}{2}$
C. 9
D. 10
E. $17 \frac{1}{2}$
14. The average of 5 distinct scores has the same value as the median of the 5 scores. The sum of the 5 scores is 420. What is the sum of the 4 scores that are NOT the median?
F. 315
G. 320
H. 336
J. 350
K. 360
15. What is the value of the expression below?

$$
||-8+4|-|3-9||
$$

A. -18
B. -2
C. 0
D. 2
E. 18
16. Which of the following expressions is equivalent to $x^{\frac{2}{3}}$ ?
F. $\frac{x^{2}}{3}$
G. $\frac{x(2)}{3}$
H. $\sqrt{x^{3}}$
J. $\sqrt[3]{x}$
K. $\sqrt[3]{x^{2}}$
17. In the standard $(x, y)$ coordinate plane, what is the slope of the line given by the equation $4 x=7 y+5$ ?
A. $-\frac{4}{7}$
B. $\frac{4}{7}$
C. $\frac{7}{4}$
D. 4
E. 7
18. For which of the following conditions will the sum of integers $m$ and $n$ always be an odd integer?
F. $m$ is an odd integer.
G. $n$ is an odd integer.
H. $m$ and $n$ are both odd integers.
J. $\quad m$ and $n$ are both even integers.
K. $m$ is an odd integer and $n$ is an even integer.
19. The lengths of the 2 legs of right triangle $\triangle A B C$ shown below are given in inches. The midpoint of $\overline{A B}$ is how many inches from $A$ ?
A. 16
B. 20
C. 21
D. 28
E. 40

20. In $\triangle D E F$, the length of $\overline{D E}$ is $\sqrt{30}$ inches, and the length of $\overline{E F}$ is 3 inches. If it can be determined, what is the length, in inches, of $\overline{D F}$ ?
F. 3
G. $\sqrt{30}$
H. $\sqrt{33}$
J. $\sqrt{39}$
K. Cannot be determined from the given information
21. Laura plans to paint the 8 -foot-high rectangular walls of her room, and before she buys paint she needs to know the area of the wall surface to be painted. Two walls are 10 feet wide, and the other 2 walls are 15 feet wide. The combined area of the 1 window and the 1 door in her room is 60 square feet. What is the area, in square feet, of the wall surface Laura plans to paint?
A. 200
B. 340
C. 360
D. 390
E. 400
22. The length of a rectangle is 5 inches longer than the width. The perimeter of the rectangle is 40 inches. What is the width of the rectangle, in inches?
F. 7.5
G. 8
H. 15
J. 16
K. 17.5
23. $8 \%$ of 60 is $\frac{1}{5}$ of what number?
A. 0.96
B. 12
C. 24
D. 240
E. 3,750
24. Armin is trying to decide whether to buy a season pass to his college basketball team's 20 home games this season. The cost of an individual ticket is $\$ 14$, and the cost of a season pass is $\$ 175$. The season pass will admit Armin to any home basketball game at no additional cost. What is the minimum number of home basketball games Armin must attend this season in order for the cost of a season pass to be less than the total cost of buying an individual ticket for each game he attends?
F. 8
G. 9
H. 12
J. 13
K. 20
25. $\frac{4.8 \times 10^{-7}}{1.6 \times 10^{-11}}=$ ?
A. $3.0 \times 10^{4}$
B. $3.0 \times 10^{-4}$
C. $3.0 \times 10^{-18}$
D. $3.2 \times 10^{18}$
E. $3.2 \times 10^{4}$
26. A circle in the standard $(x, y)$ coordinate plane has center $C(-1,2)$ and passes through $A(2,6)$. Line segment $\overline{A B}$ is a diameter of this circle. What are the coordinates of point $B$ ?
F. $(-6,-2)$
G. $(-5,-1)$
H. $(-4,-2)$
J. $(4,2)$
K. $(5,10)$
27. Which of the following expressions is a factor of $x^{3}-64$ ?
A. $x-4$
B. $x+4$
C. $x+64$
D. $x^{2}+16$
E. $x^{2}-4 x+16$
28. The average of a list of 4 numbers is 90.0 . A new list of 4 numbers has the same first 3 numbers as the original list, but the fourth number in the original list is 80 , and the fourth number in the new list is 96 . What is the average of this new list of numbers?
F. 90.0
G. 91.5
H. 94.0
J. 94.5
K. 94.8
29. The number $a$ is located at -2.5 on the number line below.


One of the following number lines shows the location of $a^{2}$. Which number line is it?
A.

B. $\begin{array}{ccccccccccccc}a^{2} \\ \mathbf{1} & 1 & 4 & & 1 & 1 & 1 & 1 & 1 & 1 \\ -8 & -6 & -4 & -2 & 0 & 2 & 4 & 6 & 8\end{array}$
C. $\begin{array}{cccccccccccccc}a^{2} \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -8 & -6 & -4 & -2 & 0 & 2 & 4 & 6 & 8\end{array}$
D.


30. Maria ordered a pizza. She ate only $\frac{2}{9}$ of it and gave the remaining pizza to her 3 brothers. What fraction of the whole pizza will each of Maria's brothers receive, if they share the remaining pizza equally?
F. $\frac{7}{9}$
G. $\frac{3}{7}$
H. $\frac{1}{3}$
J. $\frac{7}{27}$
K. $\frac{2}{27}$
31. The number 1,001 is the product of the prime numbers 7,11 , and 13 . Knowing this, what is the prime factorization of 30,030 ?
A. $3 \cdot 7 \cdot 10 \cdot 13$
B. $30 \cdot 7 \cdot 11 \cdot 13$
C. $2 \cdot 5 \cdot 7 \cdot 11 \cdot 13$
D. $3 \cdot 7 \cdot 10 \cdot 11 \cdot 13$
E. $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$

## Use the following information to answer questions 32-34.

Mikea, an intern with the Parks and Recreation Department, is developing a proposal for the new trapezoidal Springdale Park. The figure below shows her scale drawing of the proposed park with 3 side lengths and the radius of the merry-go-round given in inches. In Mikea's scale drawing, 1 inch represents 1.5 feet.

32. What is the area, in square inches, of the scale drawing of the park?
F. 448
G. 544
H. 640
J. 672
K. 1,088
33. Mikea's proposal includes installing a fence on the perimeter of the park. What is the perimeter, in feet, of the park?
A. 84
B. 88
C. 104
D. 126
E. 156
34. The length of the south side of the park is what percent of the length of the north side?
F. $112 \%$
G. $124 \%$
H. $142 \frac{6}{7} \%$
J. $175 \%$
K. $250 \%$

## Use the following information to answer

 questions 35-37.The Smith family is planning to build a 3-room cabin which consists of 2 bedrooms (BR) and 1 living room (LR). Shown below are the rectangular floor plan (left figure) and a side view of the cabin (right figure). In the side view, the roof forms an isosceles triangle ( $\triangle A B C$ ), the walls are perpendicular to the level floor $(\overline{E D}), \overline{A C} \| \overline{E D}, F$ is the midpoint of $\overline{A C}$, and $\overline{B F} \perp \overline{A C}$.


During the week the Smiths plan to roof the cabin, there is a $20 \%$ chance of rain each day.
35. Mr. Smith plans to build a 3-foot-wide walkway around the outside of the cabin, as shown in the floor plan. What will be the area, in square feet, of the top surface of the walkway?
A. 171
B. 324
C. 360
D. 396
E. 720
36. Mrs. Smith will install a ceiling fan in each room of the cabin and will place curtains over the 4 windows. Each of the ceiling fans has a price of $\$ 52.00$. The price of curtains for each small window ( S ) is $\$ 39.50$, and the price of curtains for the large window (L) is twice that for the small window. Based on this information, which of the following values is closest to the total price Mrs. Smith will pay for curtains and ceiling fans?
F. $\$ 262$
G. $\$ 302$
H. \$341
J. $\$ 354$
K. \$393
37. Mr. and Mrs. Smith plan to roof the cabin on 2 consecutive days. Assuming that the chance of rain is independent of the day, what is the probability that it will rain both days?
A. 0.04
B. 0.08
C. 0.16
D. 0.20
E. 0.40
38. Which of the following expressions, when evaluated, equals an irrational number?
F. $\frac{\sqrt{2}}{\sqrt{8}}$
G. $\frac{\sqrt{8}}{\sqrt{2}}$
H. $(\sqrt{8})^{2}$
J. $\sqrt{2} \times \sqrt{8}$
K. $\sqrt{2}+\sqrt{8}$
39. A line through the origin and $(10,4)$ is shown in the standard ( $x, y$ ) coordinate plane below. The acute angle between the line and the positive $x$-axis has measure $\theta$. What is the value of $\tan \theta$ ?
A. $\frac{\sqrt{29}}{2}$
B. $\frac{2}{\sqrt{29}}$
C. $\frac{5}{\sqrt{29}}$
D. $\frac{2}{5}$
E. $\frac{5}{2}$
40. The equation $|2 x-8|+3=5$ has 2 solutions. Those solutions are equal to the solutions to which of the following pairs of equations?
F. $2 x-5=5$
$-2 x-5=-5$
G. $\quad 2 x-8=2$
$-2 x-8=2$
H. $\quad 2 x-8=8$
$-(2 x-8)=8$
J. $\quad 2 x-8=2$
$-(2 x-8)=8$
K. $2 x-8=2$
$-(2 x-8)=2$
41. The frequency chart below shows the cumulative number of Ms. Hernandez's science students whose test scores fell within certain score ranges. All test scores are whole numbers.

| Score range | Cumulative number <br> of students |
| :---: | :---: |
| $65-70$ | 12 |
| $65-80$ | 13 |
| $65-90$ | 19 |
| $65-100$ | 21 |

How many students have a test score in the interval 71-80?
A. 1
B. 6
C. 8
D. 12
E. 13
42. The number of decibels, $d$, produced by an audio source can be modeled by the equation $d=10 \log \left(\frac{I}{K}\right)$, where $I$ is the sound intensity of the audio source and $K$ is a constant. How many decibels are produced by an audio source whose sound intensity is 1,000 times the value of $K$ ?
F. $\quad 4$
G. $\quad 30$
H. $\quad 40$
J. $\quad 100$
K. 10,000
43. Mario plays basketball on a town league team. The table below gives Mario's scoring statistics for last season. How many points did Mario score playing basketball last season?

| Type of shot | Number <br> attempted | Percent <br> successful |
| :---: | :---: | :---: |
| 1-point free throw | 80 | $75 \%$ |
| 2-point field goal | 60 | $90 \%$ |
| 3-point field goal | 60 | $25 \%$ |

A. 129
B. 190
C. 213
D. 330
E. 380
44. The graph of $y=|x-6|$ is in the standard $(x, y)$ coordinate plane. Which of the following transformations, when applied to the graph of $y=|x|$, results in the graph of $y=|x-6|$ ?
F. Translation to the right 6 coordinate units
G. Translation to the left 6 coordinate units
H. Translation up 6 coordinate units
J. Translation down 6 coordinate units
K. Reflection across the line $x=6$
45. Toby wants to find the volume of a solid toy soldier. He fills a rectangular container 8 cm long, 6 cm wide, and 10 cm high with water to a depth of 4 cm . Toby totally submerges the toy soldier in the water. The height of the water with the submerged toy soldier is 6.6 cm . Which of the following is closest to the volume, in cubic centimeters, of the toy soldier?
A. 125
B. 156
C. 192
D. 208
E. 317
46. A box in the shape of a cube has an interior side length of 18 inches and is used to ship a right circular cylinder with a radius of 6 inches and a height of 12 inches. The interior of the box not occupied by the cylinder is filled with packing material. Which of the following numerical expressions gives the number of cubic inches of the box filled with packing material?
F. $6(18)^{2}-2 \pi(6)(12)-2 \pi(6)^{2}$
G. $6(18)^{2}-2 \pi(6)(12)$
H. $18^{3}-\pi(6)(12)^{2}$
J. $\quad 18^{3}-\pi(6)^{2}(12)$
K. $18^{3}-\pi(12)^{3}$
47. A room has a rectangular floor that is 15 feet by 21 feet. What is the area of the floor in square yards ?
A. 24
B. 35
C. 36
D. 105
E. 144
48. ABC Cabs and Tary Taxicabs both have an initial fare of a whole number of dollars for 1 passenger. The fare increases a whole number of dollars at each whole number of miles traveled. The graphs below show the 1-passenger fares, in dollars, for both cab companies for trips up to 6 miles. When the fares of the 2 cab companies are compared, what is the cheaper fare for a 5-mile trip?

F. $\$ 8$
G. \$ 9
H. $\$ 10$
J. $\$ 11$
K. $\$ 12$
49. The graph of a function $y=f(x)$ consists of 3 line segments. The graph and the coordinates of the endpoints of the 3 line segments are shown in the standard $(x, y)$ coordinate plane below. What is the area, in square coordinate units, of the region bounded by the graph of $y=f(x)$, the positive $y$-axis, and the positive $x$-axis?

A. 10
B. 13
C. 14
D. 15
E. 20
50. The sum of 2 positive numbers is 151 . The lesser number is 19 more than the square root of the greater number. What is the value of the greater number minus the lesser number?
F. 19
G. 66
H. 85
J. 91
K. 121
51. The list of numbers $41,35,30, X, Y, 15$ has a median of 25 . The mode of the list of numbers is 15 . To the nearest whole number, what is the mean of the list?
A. 20
B. 25
C. 26
D. 27
E. 30
52. You are given the following system of equations:

$$
\begin{gathered}
y=x^{2} \\
r x+s y=t
\end{gathered}
$$

where $r, s$, and $t$ are integers. For which of the following will there be more than one $(x, y)$ solution, with real-number coordinates, for the system?
F. $r^{2}+4 s t>0$
G. $s^{2}-4 r t>0$
H. $r^{2}-4 s t<0$
J. $s^{2}-4 r t<0$
K. $s^{2}+4 r t<0$
53. The 3 rd and 4th terms of an arithmetic sequence are 13 and 18 , respectively. What is the 50th term of the sequence?
A. 248
B. 250
C. 253
D. 258
E. 263
54. One of the following graphs in the standard $(x, y)$ coordinate plane is the graph of $y=\sin ^{2} x+\cos ^{2} x$ over the domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. Which one?
F.

J.

G.

K.

H.

55. What is the period of the function $f(x)=\csc (4 x)$ ?
A. $\pi$
B. $2 \pi$
C. $4 \pi$
D. $\frac{\pi}{4}$
E. $\frac{\pi}{2}$
56. At the school carnival, Mike will play a game in which he will toss a penny, a nickel, and a dime at the same time. He will be awarded 3 points for each coin that lands with heads faceup. Let the random variable $x$ represent the total number of points awarded on any toss of the coins. What is the expected value of $x$ ?
F. 1
G. $\frac{3}{2}$
H. $\frac{9}{2}$
J. 6
K. 9
57. For what positive real value of $k$, if any, is the determinant of the matrix $\left[\begin{array}{ll}k & 4 \\ 3 & k\end{array}\right]$ equal to $k$ ?
(Note: The determinant of matrix $\left[\begin{array}{ll}a & b \\ c & d\end{array}\right]$ equals $a d-b c$.)
A. 3
B. 4
C. 12
D. $\sqrt{12}$
E. There is no such value of $k$.
58. Given a positive integer $n$ such that $i^{n}=1$, which of the following statements about $n$ must be true?
(Note: $i^{2}=-1$ )
F. When $n$ is divided by 4 , the remainder is 0 .
G. When $n$ is divided by 4 , the remainder is 1 .
H. When $n$ is divided by 4 , the remainder is 2 .
J. When $n$ is divided by 4 , the remainder is 3 .
K. Cannot be determined from the given information
59. For $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2},|\sin \theta| \geq 1$ is true for all and only the values of $\theta$ in which of the following sets?
A. $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$
B. $\left\{\frac{\pi}{2}\right\}$
C. $\left\{\theta \left\lvert\,-\frac{\pi}{2}<\theta<\frac{\pi}{2}\right.\right\}$
D. $\left\{\theta \left\lvert\,-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right.\right\}$
E. The empty set
60. Ray $\overrightarrow{P K}$ bisects $\angle L P M$, the measure of $\angle L P M$ is $11 x^{\circ}$, and the measure of $\angle L P K$ is $(4 x+18)^{\circ}$. What is the measure of $\angle K P M$ ?
F. $12^{\circ}$
G. $28 \frac{2}{7}{ }^{\circ}$
H. $42^{\circ}$
J. $61 \frac{1}{5}{ }^{\circ}$
K. $66^{\circ}$


Combine the totals of these columns and put in the blank for PHM in the box below.

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*Reporting Categories
PHM = Preparing for Higher Math
    \(\mathrm{N}=\) Number \& Quantity
    A = Algebra
    F = Functions
    G = Geometry
    S = Statistics \& Probability
IES = Integrating Essential Skills
MDL = Modeling
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| Number Correct (Raw Score) for: |  |
| :--- | :---: |
| Preparing for Higher Math (PHM) <br> $(\mathrm{N}+\mathrm{A}+\mathrm{F}+\mathrm{G}+\mathrm{S})$ |  |
| Integrating Essential Skills (IES) | $(35)$ |
| Total Number Correct for Mathematics Test <br> (PHM + IES) | $(60)$ |
| Modeling (MDL) <br> (Not included in total number correct for <br> mathematics test raw score) | $(28)$ |

